HYDRO STATICS

RELATIVE EQUILIBRIUM OF LIQUIDS

Relative Equilibrium of Liquids

Introduction

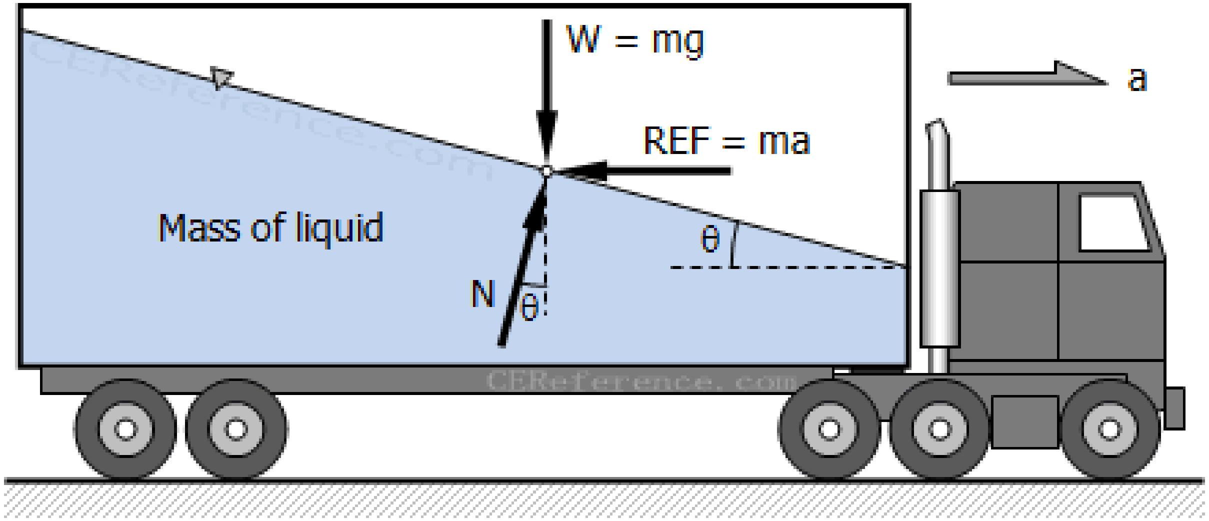
Relative equilibrium of liquid is a condition where the whole mass of liquid including the vessel in which the liquid is contained, is moving at uniform accelerated motion with respect to the earth, but every particle of liquid have no relative motion between each other. There are two cases of relative equilibrium that will be discussed in this section: linear translation and rotation. Note that if a mass of liquid is moving with constant speed, the conditions are the same as static liquid in the previous sections.

Relative Equilibrium of Liquids

Rectilinear Translation (Moving Vessel)

Horizontal Motion

If a mass of fluid moves horizontally along a straight line at constant acceleration a, the liquid surface assume an angle θ with the horizontal, see figure below.



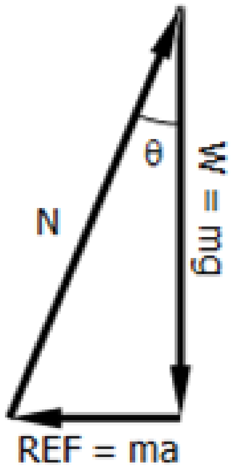
Fluid mass in horizontal accelerated motion

Relative Equilibrium of Liquids

Rectilinear Translation (Moving Vessel)

Horizontal Motion

For any value of a, the angle θ can be found by considering a fluid particle of mass m on the surface. The forces acting on the particle are the weight $W = mg$, inertia force or reverse effective force $REF = ma$, and the normal force N which is the perpendicular reaction at the surface. These three forces are in equilibrium with their force polygon:



From the force triangle

$$\tan\theta = \frac{REF}{W}$$

$$\tan\theta = \frac{ma}{mg}$$

$$\tan\theta = \frac{a}{g}$$

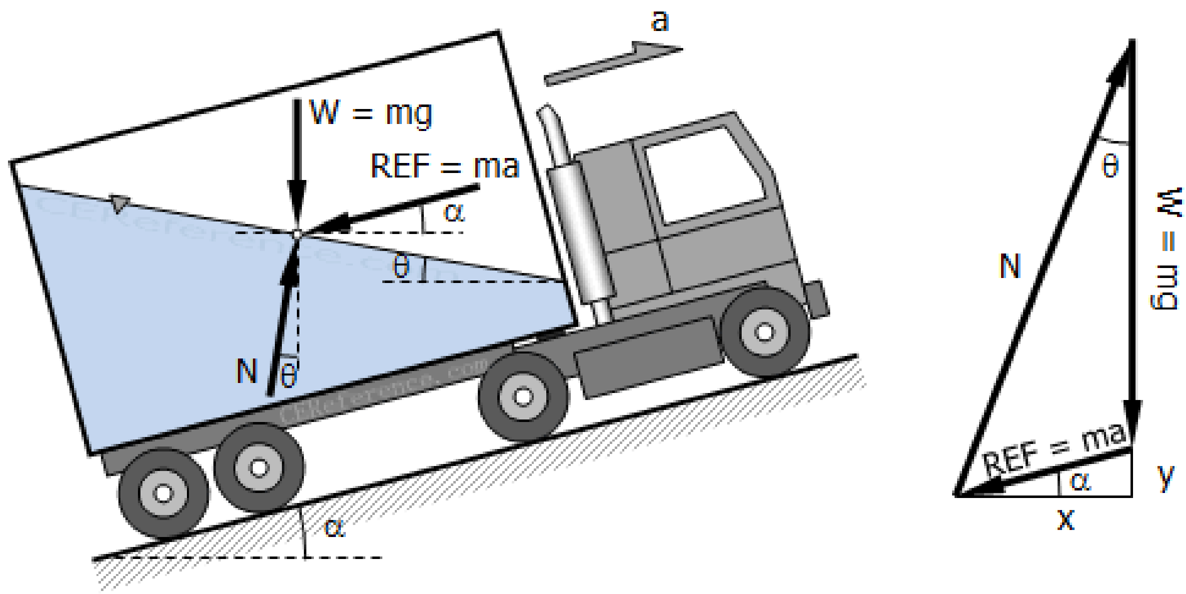
Note: The surface and all planes of equal hydrostatic pressure must be inclined at this angle θ with the horizontal.

Relative Equilibrium of Liquids

Rectilinear Translation (Moving Vessel)

Inclined Motion

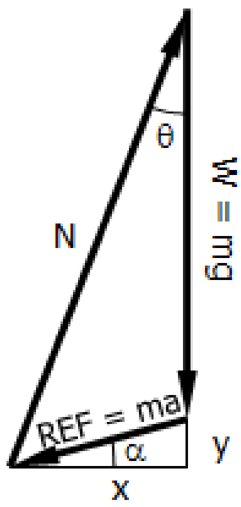
Consider a mass of fluid being accelerated up an incline α from horizontal. The horizontal and vertical components of inertia force REF would be respectively, $x = ma_h$ and $y = ma_v$.



Relative Equilibrium of Liquids

Rectilinear Translation (Moving Vessel)

Inclined Motion



From the force triangle above

$$\tan\theta = \frac{x}{W+y}$$

$$\tan\theta = \frac{ma\cos\alpha}{mg+ma\sin\alpha}$$

$$\tan\theta = \frac{a\cos\alpha}{g+a\sin\alpha}$$

but $a\cos\alpha = a_h$ and $a\sin\alpha = a_v$, hence

$$\tan\theta = \frac{a_h}{g+a_v}$$

$$\tan\theta = \frac{a_h}{g \pm a_v}$$

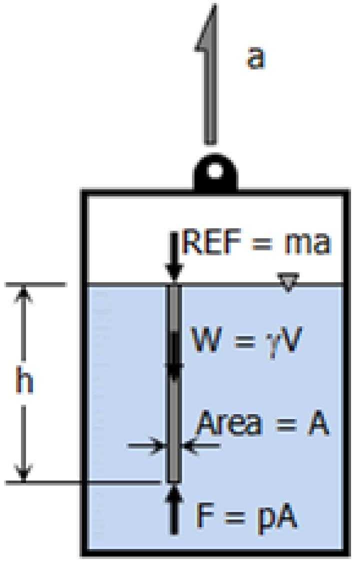
Use (+) sign for upward motion and (-) sign for downward motion.

Relative Equilibrium of Liquids

Rectilinear Translation (Moving Vessel)

Vertical Motion

The figure shown below is a mass of liquid moving vertically upward with a constant acceleration a. The forces acting to a liquid column of depth h from the surface are weight of the liquid $W = \gamma V$, the inertia force $REF = ma$, and the pressure force $F = pA$ at the bottom of the column.



Relative Equilibrium of Liquids

Rectilinear Translation (Moving Vessel)

Vertical Motion

$$\Sigma F_V = 0$$

$$F = W + REF$$

$$pA = \gamma V + ma$$

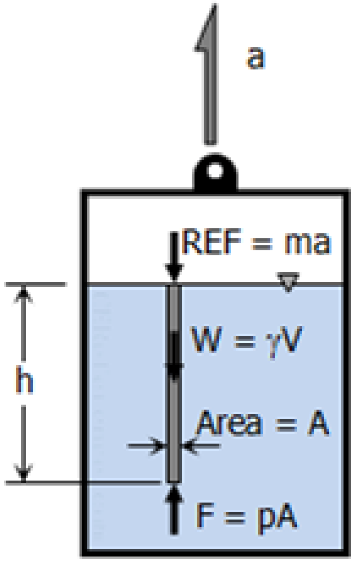
$$pA = \gamma V + \rho V a$$

$$pA = \gamma V + \frac{\gamma}{g} V a$$

$$pA = \gamma (Ah) + \frac{\gamma}{g}(Ah)a$$

$$p = \gamma h + \frac{\gamma}{g} h a$$

$$p = \gamma h \left(1 + \frac{a}{g}\right)$$



$$p = \gamma h \left(1 \pm \frac{a}{g}\right)$$

Use (+) sign for upward motion and (-) sign for downward motion. Also note that a is positive for acceleration and negative for deceleration.

Relative Equilibrium of Liquids

Problem Set 10

Problem 1

An open rectangular tank mounted on a truck is 5 *m* long, 2 *m* wide and 2.5 *m* high is filled with water to a depth of 2 *m*. Determine the following:

1.1 Maximum horizontal acceleration that can be imposed on the tank without spilling any water. (a = 1.962 m/s^2)

1.2 The accelerating force on the liquid mass. (F = 39.24 kN)

1.3 If the acceleration is increased to 6 m/s^2, how much water is spilled out? ($V_{spilled}$ = 9.78 m^3)

Relative Equilibrium of Liquids

Problem Set 10

Problem 2

A closed horizontal cylindrical tank 1.5 m in diameter and 4 m long is completely filled with gasoline (s = 0.82) and accelerated horizontally at 3 m/s^2. Determine the following:

2.1 Total force acting at the rear wall. (F_{rear} = 28.05 kN)

2.2 Total force acting at the front wall. (F_{front} = 10.66 kN)

2.3 The accelerating force on the liquid mass. (F = 17.40 kN)

Relative Equilibrium of Liquids

Problem Set 10

Problem 3

A vessel containing oil is accelerated on a plane incline 15^0 with the horizontal at 1.2 m/s^2. Determine the following:

3.1 The inclination of the oil surface when the motion is upwards. ($\Theta = 6.533^0$)

3.2 The inclination of the oil surface when the motion is downwards. ($\Theta = 6.955^0$)

Relative Equilibrium of Liquids

Problem Set 10

Problem 4

An open tank containing oil (s = 0.8) is accelerated vertically at 8 m/s^2. Determine the pressure 3 m below the surface if the motion is:

4.1 Upward with a positive acceleration. (p = 42.74 kPa)

4.2 Upward with a negative acceleration. (p = 4.34 kPa)

4.3 Downward with a positive acceleration. (p = 4.34 kPa)

4.4 Downward with a negative acceleration. (p = 42.74 kPa)